

be done on the system to rotate magnetic domains out of these directions. The local energy term associated with this interaction has been named the magneto-crystalline anisotropy energy. It has been observed that this energy term does not affect the magnitude of the saturation magnetization and depends only on the direction of the saturation magnetization relative to the local crystallographic axis.²⁶ This is a consequence of the fact that the exchange energy is much larger than the anisotropy energy and depends on the inner product of spin operators which are isotropic. Microscopically, the dominant contributing interaction to this energy is through single ion interaction with the crystal lattice.²⁶ The spin-orbit coupling prefers a colinear alignment of electron spin and orbital angular momentum while the orbital charge cloud adjusts itself in the crystal field to minimize electrostatic energy. Thus, the spin magnetic moment sees the crystal lattice through the spin orbit coupling.

The magnetoelastic energy has the same origin as the magnetocrystalline anisotropy energy. It is a consequence of the fact that the anisotropy energy is dependent on the lattice dimensions. To distort the crystal lattice in any way may change the anisotropy energy. This energy deviation from some reference lattice spacing is separated out as the magnetoelastic energy.⁴

To reemphasize, the purpose of the previous physical discussion of the microscopic origins of the various energy terms was to guide in the selection of an adequate set of thermodynamic variables. The energy will then be a function of these variables. It was concluded that the gradient of the magnetization is a reasonable choice for the continuum dependence of the exchange energy. The anisotropy energy depended on the orientation of the elemental magnetic moment within the unit cell. Hence, the magnetization vector is the logical variable. For the elastic strain, the deformation gradients will be

selected. The functional dependence of the local energy density is

$$\epsilon_{\text{LOC}}\left(\frac{\partial x_i}{\partial a_j}, \alpha_i, \frac{\partial \alpha_i}{\partial a_j}\right), \quad (2.9)$$

where x_i are the space or Eulerian coordinates and a_j are the material or Lagrangian coordinates.²⁷ It should be noted that in a purely phenomenological formulation higher order derivatives of x_i and α_i should properly be included.

2.4. Phenomenological Expression

A phenomenological expansion of the energy in a Taylor series at this point would be premature. The problem is that ϵ_{LOC} cannot be an arbitrary function of the chosen variables but must satisfy the physically obvious invariance of a rigid rotation of the mass element dm . This would restrict the form of the property tensors obtained from the Taylor expansion. To circumvent this problem, the functional dependence of ϵ_{LOC} will be recast in terms of new variables under which ϵ_{LOC} can be an arbitrary function.²⁸ This will be accomplished with a theorem due to Cauchy.²⁹

Theorem.--Any function, $f(\vec{V}_1, \dots, \vec{V}_n)$, invariant under a rigid rotation of the system of vectors, $\vec{V}_1, \dots, \vec{V}_n$, can be expressed as a function of the various quantities $\vec{V}_\alpha \cdot \vec{V}_\beta$ (α, β same or different) or $\vec{V}_\alpha \cdot \vec{V}_\beta \times \vec{V}_\gamma$ (α, β , and γ different).

ϵ_{LOC} is a function of seven vectors.

$$\epsilon_{\text{LOC}} = \epsilon_{\text{LOC}}\left(\frac{\partial \vec{r}}{\partial a_1}, \frac{\partial \vec{r}}{\partial a_2}, \frac{\partial \vec{r}}{\partial a_3}, \vec{\alpha}, \frac{\partial \vec{\alpha}}{\partial a_1}, \frac{\partial \vec{\alpha}}{\partial a_2}, \frac{\partial \vec{\alpha}}{\partial a_3}\right)$$

A sufficient choice of independent variables from Cauchy's theorem gives the functional dependence